2025

MATHEMATICS — HONOURS

Paper : DSE-B-2.3

(Advanced Mechanics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols and notations have their usual meanings unless otherwise stated.

Group - A

1. Answer the following multiple choice questions with only one correct option. choose the correct option with proper justification : 2×10

(a) A particle is placed on the top of a sphere in a gravitational field and allowed to slide without friction. Then the motion has

(i) no constraint (ii) a holonomic constraint

(iii) a non-holonomic constraint (iv) a rheonomic constraint.

(b) If p_k, q_k ($k = 1,2,3$) represent the momentum and position coordinates respectively for a system of particles, then which of the following statements is correct for the system?

(i) The configuration space is six dimensional.

(ii) The phase space is three dimensional.

(iii) The phase space is nine dimensional.

(iv) The configuration space is three dimensional.

(c) The Lagrangian of a free particle in spherical polar coordinate is given by

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\varphi}^2\sin^2\theta\right)$$

The quantity that is conserved is

(i) $mr^2\dot{\varphi}\sin^2\theta$ (ii) $mr\dot{\varphi}^2\sin^2\theta$

(iii) $mr^2\dot{\theta}^2\sin^2\theta$ (iv) $m\dot{\theta}^2\varphi\sin^2\theta$.

Please Turn Over

(d) If the Lagrangian does not depend upon time explicitly, then

(i) $\sum_{j=1}^{n} \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L$ is conserved (ii) $\sum_{j=1}^{n} q_j \frac{\partial L}{\partial q_j} - L$ is conserved

(iii) $\sum_{j=1}^{n} \dot{q}_j \frac{\partial H}{\partial \dot{q}_j} + H$ is conserved (iv) $\sum_{j} p_j \frac{\partial H}{\partial p_j} - H$ is conserved.

(e) The dynamics of a particle governed by the Lagrangian $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - kx\,\dot{x}t$ describes

(i) an undamped simple harmonic oscillator.

(ii) a damped harmonic oscillator with a time varying damping factor.

(iii) an undamped harmonic oscillator with a time dependent frequency.

(iv) a free particle.

(f) The isotropy of space leads to the law of conservation of

(i) linear momentum (ii) angular momentum

(iii) energy (iv) parity.

(g) The Lagrangian for a simple pendulum is given by

$$L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1 - \cos\theta)$$

The Poisson bracket between θ and $\dot{\theta}$ is

(i) 1 (ii) $\frac{1}{ml^2}$

(iii) $\frac{1}{m}$ (iv) $\frac{g}{l}$.

(h) Given that the linear transformation of a generalized coordinate q and the corresponding momenta p,

$$Q = q + 4\alpha p, \; P = q + 2p$$

is canonical. The value of the constant α is

(i) 1 (ii) $\frac{1}{2}$

(iii) $\frac{1}{3}$ (iv) $\frac{1}{4}$.

(i) Let A, B and C be function of phase space variables. If $\{,\}$ represents the Poisson bracket, the value of $\{A,\{B,C\}\} - \{\{A,B\},C\}$ is given by

(i) 0 (ii) $\{\{C,A\},B\}$

(iii) $\{B,\{C,A\}\}$ (iv) $\{A,\{C,B\}\}$.

(j) The transformation of Lagrangian to Hamiltonian is done by

(i) Laplace transformation (ii) Legendre transformation

(iii) Lorentz transformation (iv) Galilean transformation.

Group - B

Unit - 1

(Marks : 10)

2. Answer ***any two*** questions :

(a) A bead of mass m is constrained to move on a straight frictionless wire AB, fixed at a point A on a vertical axis OA and rotating about OA with constant angular velocity ω. Set-up the Lagrangian and find the Lagrange's equation of motion. 4+1

(b) State D'Alembert's principle. Derive Lagranges's equations of motion from it for conservative systems. 1+4

(c) For a particle moving under a central force, construct Lagrange's equations of motion. 5

(d) The Lagrangian for a system with 2 degrees of freedom is given by $L = \dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_1\dot{q}_2 - q_1^2 - q_2^2$. Use Lagrange's equation to find q_1 and q_2. 5

Unit - 2

(Marks : 15)

3. Answer ***any three*** questions :

(a) State the Hamilton's principle and derive the Hamilton's canonical equations of motion from it. 1+4

(b) Consider a planet of mass m orbiting around the sun under the inverse square law of attraction $\frac{\mu m}{r^2}, \mu > 0$, then find the Hamiltonian of the system and also write down the Hamilton's equations of motion. 5

Please Turn Over

(c) (i) For two-dimensional harmonic oscillator, Lagrangian is given by

$$L = \frac{1}{2}m\left(\dot{r}^2 + \dot{r}^2\dot{\theta}^2\right) - \frac{1}{2}kr^2$$

Find the Hamilton's equations of motion.

(ii) If the Lagrangian of a dynamical system is $L = \dot{q}_1^2 + \dot{q}_2^2 + k_1 q_1^2$, then find the Hamiltonian of the system. 2+3

(d) A particle of mass m is constrained to move on the surface of a cylinder defined by $x^2 + y^2 = R^2$. It is subjected to a force $F = -Kr$, proportional to its distance from the origin and directed to it. Using the Hamiltonian, write down the canonical equation of motion and show that motion in z-direction is simple harmonic. 4+1

(e) For the Hamiltonian $H = p_1 q_1 - p_2 q_2 - aq_1^2 + bq_2^2$, solve the Hamilton's equation of motion and prove that $(p_2 - bq_2)/q_1$ = constant and $q_1 q_2$ = constant, where a, b are constants and q_1, q_2 are generalized co-ordinates, p_1, p_2 are corresponding generalized momentum. 3+1+1

Unit - 3

(Marks : 10)

4. Answer ***any two*** questions :

(a) State and prove Liouville's theorem. 5

(b) The Hamiltonian for a system described by the generalized co-ordinate x and generalized momentum p is

$$H = \alpha x^2 p + \frac{p^2}{2(1+2\beta x)} + \frac{1}{2}\omega^2 x^2$$

where α, β, ω are constants. Find the corresponding Lagrangian of the system. 5

(c) Derive the Hamilton's equations of motion in spherical polar coordinates. 5

(d) Lagrangian of an anharmonic oscillator of unit mass is $L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega^2 x^2 - \alpha x^3 + \beta . x . \dot{x}^2$, α and β being constants. Find the Hamiltonian and deduce the canonical equations of motion. 5

Unit - 4

(Marks : 10)

5. Answer ***any two*** questions :

(a) (i) Show that if a function F does not depend on time explicitly and is a constant of motion, then its Poisson bracket with the Hamiltonian vanishes.

(ii) Write the equations of motion in Poisson bracket form. 3+2

(b) (i) Show that the transformation $q = P^2 + Q^2, p = \frac{1}{2}\tan^{-1}\left(\frac{P}{Q}\right)$ is canonical.

(ii) Find the canonical transformations defined by generating function

$F_1(q, Q) = qQ - \frac{1}{2}m\omega q^2 - Q^2 / 4m\omega.$ 2+3

(c) (i) If $[\varphi, \psi]$ be the Poisson bracket of φ and ψ, then prove that $\frac{\partial}{\partial t}[\varphi, \psi] = \left[\frac{\partial \varphi}{\partial t}, \psi\right] + \left[\varphi, \frac{\partial \psi}{\partial t}\right]$.

(ii) The Poisson bracket of q and $f(p)$ is given by $[q, f(p)] = f(p)$. Find $f(p)$. 3+2

(d) What do you understand by Hamilton's principal function? Use it to solve the problem of a particle falling freely under the action of gravity. 5