2025

MATHEMATICS — HONOURS

Paper : DSE-A-2.2

(Mathematical Modelling)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group – A

1. Answer all questions : 2×10

(Each question has four alternatives of which **one** is correct. Choose the correct one with proper justification. **One mark** is for correct answer and **one mark** is for proper justification.)

(a) The set of regular singular points of the ordinary differential equation

$$x\left(x^3+x^2-x-1\right)^2\frac{d^2y}{dx^2}-\left(x^4+3x^3+3x^2+x\right)\frac{dy}{dx}+\left(x^4-5x^2+4\right)y=0 \text{ is}$$

(i) $\{0, -1\}$ (ii) $\{0\}$

(iii) $\{0, 1, -1\}$ (iv) $\{0, 1\}$.

(b) The value of the limit $\lim_{x\to 0}\frac{J_n(x)}{x^n}$ when $n \in N$ is

(i) $\frac{n!}{2^n}$ (ii) $\frac{1}{2^n n!}$

(iii) $\frac{2^n}{n!}$ (iv) $\frac{1}{2^n}$.

(c) Using middle square method with seed $x_0 = 2025$, the 4th random number generated is

(i) 1006 (ii) 0120

(iii) 0144 (iv) 0207.

(d) Laplace transform of the function $e^{-5t}t^3$ is

(i) $\frac{6}{(p+5)^4}$ (ii) $\frac{3}{(p+5)^4}$

(iii) $\frac{6}{(p-5)^4}$ (iv) $\frac{5}{(p-3)^5}$.

(e) The Laplace transform of $f(t) = 3e^{2t} + 4e^{-3t}$ is

(i) $\frac{7s-1}{s^2-s+6}$ (ii) $\frac{7s+1}{s^2+s-6}$

(iii) $\frac{7s-1}{s^2+s+6}$ (iv) $\frac{7s+1}{s^2+s+6}$.

(f) If arrival rate is 3 customers per day and service rate is 5 customers per day for a (M/M/I : ∞|FIFO) queueing model, then the average number of customer in the system at Q certain day is

(i) 1.5 (ii) 2

(iii) 3 (iv) 5.

(g) If arrival and service rates are α and β respectively, then idle rate is given by

(i) $1-\frac{1}{\alpha}$ (ii) $1-\frac{\alpha}{\beta}$

(iii) $1-\frac{1}{\beta}$ (iv) $1-\frac{\beta}{\alpha}$.

(h) $P_n(x)$ is Legendre polynomial of degree n, then $\int_{-1}^{+1} (P_3(x))^2 dx$ is

(i) $\frac{3}{2}$ (ii) 1

(iii) 4 (iv) $\frac{4}{3}$.

(i) Number of extreme points of the convex set $S = \{(x, y) \| x | \leq 1, y \leq 2\}$

(i) 4 (ii) 3

(iii) 2 (iv) 6.

(j) In Monte Carlo simulation of a fair die the random numbers x_n are generated between 0 and 3. Then which one denotes event of obtaining 1 in die?

(i) $0 < x_n < \frac{1}{6}$ (ii) $0 < x_n < 1$

(iii) $\frac{1}{3} < x_n < 1$ (iv) $0 < x_n < \frac{1}{2}$.

Group - B

Unit - I

2. Answer ***any two*** questions :

(a) (i) Solve the initial value problem $\frac{d^2y}{dt^2}-\frac{dy}{dt}-2y=18e^{-t}\sin 3t,\ y(0)=0,\ y'(0)=3$ using Laplace transform method.

(ii) Use convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s-2)(s^2+1)}\right\}$. 5+5

(b) (i) Prove that $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)}=\sum_{n=-\infty}^{\infty}t^nJ_n(x)$.

(ii) Prove that $\left(1-2xt+t^2\right)^{-\frac{1}{2}}=\sum_{n=0}^{\infty}t^nP_n(x),\ |t|<1, |x|\le 1$. 5+5

(c) (i) Find the solution in series of $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+x^2y=0$ about $x=0$.

(ii) If $P_n(x)$ is Legendre polynomial of degree n, prove that

$$xP_n(x)=\frac{1}{2n+1}\left[(n+1)P_{n+1}(x)+nP_{n-1}(x)\right].$$ 5+5

(d) (i) Evaluate : $L^{-1}\left\{\tan^{-1}\left(\frac{a}{p}\right)\right\}$.

(ii) Find the Laplace transform of $f(t)=\begin{cases}0 & 0<t<\frac{\pi}{2}\\ \sin t & t>\frac{\pi}{2}\end{cases}$. 5+5

Unit - II

3. Answer *any five* questions :

(a) Solve the following LPP by graphical method :

Minimize $z = 4x_1 + x_2$

subject to $x_1 + 2x_2 \le 3$

$4x_1 + 3x_2 = 6$

$3x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0.$ 5

(b) Solve the following L.P.P. by algebraic method to get the optimal solution :

Maximize $z = 25x_1 + 30x_2$

subject to $20x_1 + 30x_2 \le 690$

$5x_1 + 4x_2 \le 120$

$x_1, x_2 \ge 0.$ 5

(c) Describe Linear Congruence method to generate random numbers mentioning its drawback. Determine 5 random numbers using the formula $x_{n+1} = (4x_n + 3) \bmod (50)$ with need $x_0 = 156$. (2+1)+2

(d) For a M | M | 1 : ∞ | FIFO queueing model a steady state, find the probability of n units in the queueing system. 5

(e) A T.V. repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8-hour day.

(i) What is the repairman's expected idle time each day?

(ii) What is the average queue length?

(iii) Also find the average number of jobs in the system. 2+1+2

(f) The automobile company manufacture around 150 scooters. The daily production varies from 146 to 154 depending upon the availability of raw materials and other working conditions :

Production (per day)	146	147	148	149	150	151	152	153	154
Probability	0.04	0.09	0.12	0.14	0.11	0.10	0.20	0.12	0.08

The finished scooters are transported in a lorry accommodating 150 scooters. Using following random numbers :

80, 81, 76, 75, 64, 43, 18, 26, 10, 12, 65, 68, 69, 61, 57.

Simulate the process to find out the average number of scooters waiting in the factory. 5

(g) A small harbour has unloading facility for ships. Only one ship can be unloaded at any one time. Time span between the arrival of successive ships and unloading time of the five ships are given in the table below :

Description	Ship-1	Ship-2	Ship-3	Ship-4	Ship-5
Time between successive ships	20	30	15	120	20
Unloading time	55	45	60	75	80

If all the time spans are given in minutes, then determine

(i) Mean waiting time for the ships

(ii) Average unload time for the ships

(iii) Average time spent by the ships in the harbour

(iv) Total idle time for the harbour. 5

(h) Consider the following LPP :

Maximize $z = 5x_1 + 3x_2$

subject to $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0.$

After finding the optimal solution, obtain the range of C_1 to get same optimal solution, where C_1 is the cost coefficeint of variable x_1. 5