2025

MATHEMATICS — HONOURS

Paper : CC-14

(Numerical Methods)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meaning.

Unit - 1

1. Each of the following questions has four possible answers of which exactly one is correct. Choose the correct alternative : 1×10

(a) If the value of $e = 2.71828$ is replaced by 2.71937, then percentage error is

(i) 0.04% (ii) 0.05%

(iii) 4% (iv) $\frac{1}{2}$%.

(b) Taking step length $h = \frac{1}{2}$, what will be the value of $\Delta^6\left[\left(1-\frac{x}{3}\right)\left(1-\frac{4}{5}x^2\right)\left(1-\frac{x^2}{3}\right)\right]$?

(i) 1 (ii) −1

(iii) 0 (iv) 6!.

(c) Suppose in an interpolation method we use argument set $\{-2, 0, 1\}$ and get corresponding interpolating polynomial as $\frac{1}{4}x^3 + x^2 + 1$. Which method is used here?

(i) Newton's interpolation

(ii) Lagrange's interpolation

(iii) Divided Difference interpolation

(iv) Hermite interpolation.

(d) The expression for the first derivative from Newton's forward interpolation formula at $x = x_0$ is given by

(i) $f'(x_0) = h\left[\Delta y_0 + \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 + \frac{1}{4}\Delta^4 y_0 + ...\right]$

(ii) $f'(x_0) = h\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + ...\right]$

(iii) $f'(x_0) = \frac{1}{h}\left[\Delta y_0 + \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 + \frac{1}{4}\Delta^4 y_0 + ...\right]$

(iv) $f'(x_0) = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + ...\right]$

(e) If $f(x) = 3x^5 + 5$, then which one gives exact result when applied to $\int_1^2 f(x)dx$?

(i) The Trapezoidal rule (ii) The Simpson's one-third rule

(iii) The Weddle's rule (iv) The Simpson's $\frac{3}{8}$th rule.

(f) If a quadrature formula $\frac{3}{2}f\left(-\frac{1}{3}\right) + kf\left(\frac{1}{3}\right) + \frac{1}{2}f(1)$, that approximates $\int_{-1}^{1} f(x)dx$, is found to be exact for quadratic polynomials, then the value of k is

(i) 2 (ii) 1

(iii) 0 (iv) −2.

(g) Order of convergence of Newton-Raphson method, when we find out the value of $\sqrt[5]{3}$, correct up to four decimal places, is

(i) 1 (ii) 2

(iii) 3 (iv) 4.

(h) For which of the functions $\phi(x)$ below, is $\alpha = \sqrt{5}$ a fixed point?

(i) $\phi(x) = \frac{x}{\sqrt{5}}$ (ii) $\phi(x) = \sqrt{5x}$

(iii) $\phi(x) = x^2 - 4x$

(iv) $\phi(x) = 1 + \dfrac{4}{x+1}$.

(i) Which of the following matrices is strictly diagonally dominant?

(i) $\begin{bmatrix} -6 & 0 & 3 \\ 7 & 8 & -2 \\ 1 & -1 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} -6 & 0 & 3 \\ 5 & 8 & -2 \\ 1 & -1 & 2 \end{bmatrix}$

(iii) $\begin{bmatrix} -6 & 0 & 3 \\ 5 & 8 & -2 \\ 1 & -1 & 3 \end{bmatrix}$

(iv) $\begin{bmatrix} -6 & 0 & 3 \\ 1 & 2 & -2 \\ 1 & -1 & 8 \end{bmatrix}$.

(j) The value of $y(2.5)$ obtained by solving the initial value problem

$$\frac{dy}{dx} = \frac{y-2}{x+2},\ y(2) = -1,$$

by Euler's method taking $h = 0.5$, is

(i) 0.25

(ii) -0.625

(iii) -1

(iv) -1.375.

Unit - 2

Answer *any one* question.

2. (a) Prove that $\Delta \log f(x) = \log\left[1 + \dfrac{\Delta f(x)}{f(x)}\right]$.

(b) If $f(x) = x^3$, show that the third order divided difference $f[x_0, x_1, x_2, x_3]$ is a constant. 2+3

3. Derive Lagrange's interpolation formula for $(n+1)$ data (x_i, y_i), $i = 0, 1, \dots, n$. 5

Unit - 3

Answer *any two* questions.

4. Deduce Newton's forward differentiation formula from Newton's forward interpolation formula for $(n+1)$ data (x_i, y_i), $i = 0, 1, \dots, n$ (Give at least four terms). 5

5. Derive Simpson's $\frac{1}{3}$rd rule using Newton's interpolation polynomial. Define degree of precision of a quadrature formula. What is the degree of precision of Simpson's $\frac{1}{3}$ rule? 3+1+1

6. Deduce Trapezoidal rule from Newton-Cote's formula and write its error term also. Find the largest value of spacing h, so that error in computing $\int_{1.8}^{3.4} e^x dx$ using Trapezoidal rule is less than or equal to 0.5×10^{-5}. 3+2

7. Find the values of A, B, C such that $\int_0^h f(x)dx = h\left[Af(0) + Bf\left(\frac{h}{2}\right) + Cf(h)\right]$ is exact for polynomials $f(x)$ of degree up to 2. 5

Unit - 4

Answer *any two* questions.

8. Describe the fixed point iteration method and state a sufficient condition of convergence for this method. Write down the fixed point iteration scheme for the equation $x^3 + x - 1 = 0$ asserting the convergence of the scheme. 3+2

9. Using Newton-Raphson method obtain the numerical scheme for finding $\sqrt{N}$, where N is a positive real number. Hence apply the scheme to $N = 18$ to obtain result correct to two decimals. 3+2

10. Define the order of convergence of an iteration method for computing a simple real root of a non-linear equation $f(x) = 0$. Find the order of convergence of Newton-Raphson method for computing a simple real root of a non-linear equation $f(x) = 0$. 1+4

11. Derive the secant method for finding a real simple root of an equation $f(x) = 0$. 5

Unit - 5

Answer *any two* questions.

12. Describe the Gauss-Seidel iteration method for solving a system of linear equations. State the sufficient condition of convergence for the method. What is the advantage of this method over Gauss elimination method? 3+1+1

13. Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

using Crout's LU decomposition method. 5

14. Describe the Gauss-Jordan elimination method for numerical solution of a system of n linear equations in n unknowns $AX = b$. How does it differ from Gauss elimination method? 3+2

15. Describe the power method for finding the largest eigenvalue and the corresponding eigenvector of the square matrix A of order n. 5

Unit - 6

Answer ***any one*** question.

16. Explain Euler's method to solve the differential equation $\frac{dy}{dx} = f(x, y)$ with initial condition $y(x_0) = y_0$. Give its geometrical interpretation. 5

17. Obtain the Picard's iteration formula for the initial value problem $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$. Use it to the problem $\frac{dy}{dx} = y - x^2, y(0) = 1$ up to the third approximation. 2+3

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Unit - 6

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