2024

MATHEMATICS — HONOURS

Paper : DSE-B-2.1, DSE-B-2.2 and DSE-B-2.3

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

DSE-B-2.1

(Point Set Topology)

Full Marks : 65

Throughout the question, $\mathbb{R}$, $\mathbb{Z}$ and $\mathbb{N}$ denote respectively the set of real numbers, the set of integers and the set of natural numbers.

1. Answer all multiple choice questions. For each question, **1** mark for choosing correct option and **1** mark for justification. 2×10

(a) Let X be an infinite set. Then the topology on X generated by the base $\{\{x\} : x \in X\}$ is

(i) stronger than the co-finite topology on X.

(ii) T_1 but not T_2.

(iii) compact.

(iv) connected.

(b) Let β_1 be the collection of all bounded open intervals of $\mathbb{R}$ and β_2 be the collection of all half open intervals of $\mathbb{R}$ of the form $(a, b]$. Then which of the following is TRUE?

(i) β_1 and β_2 both form a base giving the same topology on $\mathbb{R}$.

(ii) The topology generated by β_1 is finer than the topology generated by β_2 on $\mathbb{R}$.

(iii) The topology generated by β_2 is finer than the topology generated by β_1 on $\mathbb{R}$.

(iv) The topologies generated by β_1 and β_2 on $\mathbb{R}$ are non-comparable.

(c) Let X be a non-empty set endowed with the topology $\{A \subseteq X : p \in A\} \cup \{\phi\}$, where $p \in X$. Then the derived set of $\{p\}$ is

(i) ϕ (ii) $\{p\}$

(iii) $X \setminus \{p\}$ (iv) X.

(d) $Y = (0, 1)$ is a subspace of $\mathbb{R}$ with usual topology. The subset $\left(0, \frac{1}{2}\right]$ of Y is

(i) an open set in Y. (ii) a closed set in Y.

(iii) a clopen set in Y. (iv) a dense set in Y.

(e) Let $\tau = \{\phi, X, \{1, 2\}\}$ be a topology on $X = \{1, 2, 3\}$. A sequence $\{x_n\}$ in X, where $x_n = 1$, $\forall n \in \mathbb{N}$

(i) converges to a unique point 1.

(ii) does not converge.

(iii) converges to a unique point 2.

(iv) converges to each and every point in X.

(f) Let $f : (\mathbb{R}, \tau_u) \to (\mathbb{R}, \tau_u)$ be a continuous map (where τ_u denotes the usual topology on $\mathbb{R}$) and $Z(f) = \{x \in \mathbb{R} : f(x) = 0\}$. Then

(i) $Z(f)$ is a closed set. (ii) $Z(f)$ is a connected set.

(iii) $Z(f)$ is a compact set. (iv) $Z(f)$ is an open set.

(g) Which of the following topological space is compact?

(i) $\mathbb{R}$ with co-finite topology. (ii) $\mathbb{R}$ with usual topology.

(iii) $\mathbb{R}$ with discrete topology. (iv) None of these.

(h) A connected subset U of the real line $\mathbb{R}$ with more than one point must be

(i) an open set. (ii) a closed set.

(iii) a compact set. (iv) an uncountable set.

(i) The components of $\mathbb{R}$ with discrete topology are

(i) singleton sets. (ii) finite sets with at least two points.

(iii) countably infinite sets. (iv) uncountable sets.

(j) The Sorgenfrey line is

(i) compact. (ii) connected.

(iii) first countable. (iv) T_1 but not T_2.

Unit - 1

(Marks : 20)

Answer ***any four*** questions.

2. (a) Let $\mathbb{N}$ be the set of natural numbers and $E_m = \{m, m+1, m+2, ...\}$, $m \in \mathbb{N}$. Then prove that $\tau = \{E_m : m \in \mathbb{N}\} \cup \{\phi\}$ is a topology on $\mathbb{N}$.

(b) Prove or disprove : union of any collection of topologies on a non-empty set X is a topology on X. 3+2

3. (a) Show that the collection $\mathcal{B} = \{[a, b) : a < b \text{ and } a, b \in \mathbb{Q}\}$ is a basis that generates a topology on $\mathbb{R}$ that is different from the lower limit topology on $\mathbb{R}$. ($\mathbb{Q}$ is the set of rational numbers).

(b) Prove or disprove : The subspace topology on $\mathbb{Z}$ is the discrete topology on $\mathbb{Z}$, where $\mathbb{Z}$ is the subset of $\mathbb{R}$ endowed with usual topology. 3+2

4. (a) (X, τ) is a topological space with a basis $\mathcal{B}$ and $A \subseteq X$. Prove that $x \in \overline{A}$ if and only if every element B in $\mathcal{B}$ containing x intersects A.

(b) Prove that every infinite subset of a co-finite topological space (X, τ) is dense in X. 3+2

5. Let (X, d) be a metric space and A be any non-empty subset of X. Prove that $\overline{A} = \bigcap \left\{ B\left(A, \frac{1}{n}\right) : n \in \mathbb{N} \right\}$,

where $B(A, r) = \bigcup_{x \in A} B(x, r)$ for any $r > 0$ and $B(x, r) = \{ y \in X : d(x, y) < r \}$.

Hence prove that every open set in a metric space is a countable union of closed sets. 3+2

6. (a) (X, τ_1), (Y, τ_2) and (Z, τ_3) are three topological spaces and $f_1 : X \to Y$, $f_2 : X \to Z$ are two functions. Prove that the function $f : X \to Y \times Z$ defined by $f(x) = (f_1(x), f_2(x))$ is continuous if and only if f_1 and f_2 are continuous.

(b) (X, τ_1) and (Y, τ_2) are two topological spaces and $f : X \to Y$ is a continuous function. Prove that if x is a limit point of A in X, then $f(x)$ is a limit point of $f(A)$ in Y. 3+2

7. (a) Let (X_1, τ_1) and (X_2, τ_2) be two topological spaces. Prove that the family $\mathcal{B} = \{U \times V : U \in \tau_1, V \in \tau_2\}$ is a base for some topology on the cartesian product $X_1 \times X_2$.

(b) Prove that the projection mapping $\pi_1 : X \times Y \to X$ defined by $\pi_1(x, y) = x \ \forall \ (x, y) \in X \times Y$ is open and continuous, where $X \times Y$ is the product space of topological spaces (X, τ) and (Y, σ). 3+2

8. (a) Let (X, d) be a metric space. Prove that the metric $d : X \times X \to \mathbb{R}$ is a continuous function, where $X \times X$ is endowed with the product topology and $\mathbb{R}$ is endowed with the usual topology.

(b) Prove that, $\mathbb{R}$ with usual topology is homeomorphic with the subspace $(-1, 1)$ of the real line. 3+2

Unit - 2

(Marks : 10)

Answer ***any two*** questions.

9. (a) Prove that in a first countable space (X, Y), each $x \in X$ has a countable local base $\{V_n : n \in \mathbb{N}\}$ such that $V_{n+1} \subseteq V_n, \forall n \in \mathbb{N}$.

(b) Prove or disprove : an uncountable set X with co-finite topology is first countable. 2+3

10. Suppose (X, τ) be a first countable space and $f : X \to Y$ be a map, where (Y, σ) is a topological space. Prove that f is continuous at $c \in X$ if and only if whenever a sequence $\{x_n\} \subseteq X$ converges to c, the sequence $\{f(x_n)\}$ converges to $f(c) \in Y$. 5

11. (a) Let (X, τ) be a topological space such that the diagonal $\{(x, x) \in X \times X : x \in X\}$ is a closed set in the product space $X \times X$. Prove that (X, τ) is a T_2 space.

(b) Prove that a topological space (X, τ) is T_1 if every singleton subset $\{x\}$ of X is closed. 3+2

12. (a) Prove that the limit of a convergent sequence is unique in a T_2 space.

(b) Prove that product space of two T_1 spaces is a T_1 space. 2+3

Unit - 3

(Marks : 15)

Answer *any three* questions.

13. (a) Prove that a topological space X with co-countable topology is compact if and only if X is finite.

(b) Prove or disprove : in the topological space $\mathbb{R}$ with usual topology, the set $\left\{\frac{1}{n} : n \in \mathbb{Z} \setminus \{0\}\right\} \cup \{0\}$ is compact. 3+2

14. (a) Prove that a closed subset of a compact topological space is compact.

(b) Let f be a one-one continuous mapping from a compact topological space (X, τ) onto a T_2-space (Y, τ'), then show that X and Y are homeomorphic. 2+3

15. (a) Prove that in any topological space (X, τ), closure of any connected subset of X is also a connected subset.

(b) Show that there does not exist a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $f(\mathbb{Q}) \subseteq \mathbb{R} \setminus \mathbb{Q}$ and $f(\mathbb{R} \setminus \mathbb{Q}) \subseteq \mathbb{Q}$, where $\mathbb{R}$ is endowed with the usual topology. 3+2

16. (a) Prove that a continuous map from a countable connected space to the real line $\mathbb{R}$ is constant.

(b) Show that if (X, τ) is disconnected and τ_0 is finer than τ, then (X, τ_0) is disconnected. 3+2

17. (a) Show that a topological space (X, τ) is disconnected if and only if there exists a continuous mapping from X onto the discrete two point space $\{0, 1\}$.

(b) Prove or disprove : the boundary of any non-empty proper subset of a connected topological space X is non-empty. 3+2

DSE-B-2.2

(Astronomy and Space Science)

Full Marks : 65

Notations /symbols have their usual meaning.

1. Answer all the following multiple choice questions. For each question **1 mark** for choosing correct option and **1 mark** for justification : 2×10

(a) Antares appears to be 7.87 times brighter than η Scorpii. The magnitude of Antares is 1.2. Then the magnitude of η Scorpii is

(i) 3.44 (ii) 3.54

(iii) 3.64 (iv) 3.74.

(b) The limits of declination for stars which are circumpolar at a place of latitude 30°(N) are

(i) 60° and 90° (ii) 30° and 90°

(iii) 30° and 60° (iv) none of these.

(c) The solar hour angle at 2.00 pm is

(i) 15° (ii) -15°

(iii) 30° (iv) -30°.

(d) The microwave background radiation has a spectrum which peaks at a wavelength of 1.1 mm and is identical in shape to that of a black body of temperature 2.7 K. At what wavelength will the spectrum of the star Sirius with A (temperature 9940 K) peak?

(i) 9036 nm (ii) 335 nm

(iii) 299 nm (iv) 34 nm.

(e) The horizontal parallax of the moon is $57'$ and her angular diameter is $31'5''$. The radius of the Earth is 6400 km. The diameter of the moon in kilometers is

(i) 3495 (ii) 3493

(iii) 3490 (iv) 3489.

(f) The distance of Saturn from the Sun is 9.54 times that of the Earth. The interval between two successive conjunctions of the Earth and Saturn is nearly

(i) 376 days (ii) 377 days

(iii) 378 days (iv) 375 days.

DSE-B-2.2

(Astronomy and Space Science)

Full Marks : 65

Notations / symbols have their usual meaning.

1. Answer all the following multiple choice questions. For each question **1 mark** for choosing correct option and **1 mark** for justification : 2×10

(a) Antares appears to be 7.87 times brighter than η Scorpii. The magnitude of Antares is 1.2. Then the magnitude of η Scorpii is

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(iii) 3.64 (iv) 3.74.

(b) The limits of declination for stars which are circumpolar at a place of latitude 30°(N) are

(i) 60° and 90° (ii) 30° and 90°

(iii) 30° and 60° (iv) none of these.

(c) The solar hour angle at 2.00 pm is

(i) 15° (ii) –15°

(iii) 30° (iv) –30°.

(d) The microwave background radiation has a spectrum which peaks at a wavelength of 1.1 mm and is identical in shape to that of a black body of temperature 2.7 K. At what wavelength will the spectrum of the star Sirius with A (temperature 9940 K) peak?

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(i) 3495 (ii) 3493

(iii) 3490 (iv) 3489.

(f) The distance of Saturn from the Sun is 9.54 times that of the Earth. The interval between two successive conjunctions of the Earth and Saturn is nearly

(i) 376 days (ii) 377 days

(iii) 378 days (iv) 375 days.

(g) The temperature of a star with the peak of its black body spectrum at a wavelength of 271 nm is approximately

(i) 2.0×10^4 K (ii) 3.0×10^4 K

(iii) 1.50×10^4 K (iv) 1.07×10^4 K.

(h) The redshift of a nearby galaxy is 0.01. If the Hubble constant is 73 $\text{km}s^{-1}\text{Mpc}^{-1}$, then the distance of the galaxy in Mpc is

(i) 7.3 Mpc (ii) 21.9 Mpc

(iii) 730 Mpc (iv) 41.1 Mpc.

(i) A star of magnitude +4 lies at a distance 100 pc. The absolute magnitude of the star is

(i) +4.0 (ii) −1.0

(iii) +9.0 (iv) +1.49.

(j) A satellite measures the parallax angle of a star as 0.002 arc second. The distance of the star is

(i) 500 pc (ii) 300 pc

(iii) 900 pc (iv) none of these.

Unit - 1

2. Answer ***any one*** question :

(a) Write down the sine and cosine formulas for spherical trigonometry. Using it, calculate for a given spherical triangle the angle A and side C, where $a = 75°$, $b = 45°$, $c = 90°$. 2½+2½

(b) What is meant by Midnight Sun? Neglecting the eccentricity of the Earth's orbit, prove that at a place within the arctic circle the sun will be above the horizon for $\dfrac{365\frac{1}{4}}{\pi}\cos^{-1}\left(\dfrac{\cos\phi}{\sin\omega}\right)$ days. Here ϕ represents the latitude of the place and ω represents the obliquity of the ecliptic. 1+4

Unit - 2

3. Answer ***any one*** question :

(a) What is Compton effect? Derive the expression for the change in wavelengths of the incident and scattered photons in terms of the scattering angle. 1+4

(b) If the polarization angle of a medium is 45°, find the index of refraction of the medium. Compare the brightness of images of the Moon produced by two telescopes with $f = 200$ cm, $a = 40$ cm and $f = 500$ cm, $a = 80$ cm. 2+3

Unit - 3

4. Answer ***any two*** questions :

(a) With the aid of a diagram, describe the Moving Cluster method for determining the distance, d, to a nearby star cluster and show that $d = \frac{V_R \tan(\theta)}{4.74\mu}$, where V_R is the radial velocity of the cluster in km s^{-1}, θ is the angle to the convergence point in degrees and μ is the proper motion of the cluster in arcsec per year. 5

(b) Define Elongation and Conjunction of a planet. Show that the elongation θ of Venus when it is brightest is given by the equation $3\cos^2\theta + 4k\cos\theta - 4 = 0$, where k is the ratio of its distance from the sun to that of the Earth. 2+3

(c) Calculate the time taken by light to travel to the Earth from a star with parallax $p = 0''.90$. A star is 6.25 light years away from our solar system. How much is the distance in terms of parsecs? 3+2

(d) The luminosity of a star of apparent magnitude +4 is 150 times that of the sun and its proper motion is $0''.30$. Find the tangential velocity of this star. Derive the distance-modulus relation. 2+3

Unit - 4

5. Answer ***any one*** question :

(a) What are interstellar cloud collisions? Discuss Kahn's model of interstellar cloud collisions and calculate the rate of kinetic energy dissipated by the process. 1+4

(b) What do you mean by Jean's instability? Discuss the Jean's mass criterion for a self-gravitating isothermal gas cloud. 2+3

Unit - 5

6. Answer ***any two*** questions :

(a) Briefly describe the morphological classification of galaxies. State morphological type of our galaxy using Hubble's scheme. What is the length of our galaxy's optical diameter? 3+1+1

(b) What are the principal difficulties that are faced in deriving the correct value of the Hubble constant H? Give a physical interpretation of the Hubble constant. 3+2

(c) Draw a diagram of the rotation curve of our galaxy and obtain a polynomial in the radial distance 'r' that fits the rotation curve fairly well. 2+3

(d) Discuss how from the intensity of 21 cm line radiation the neutral hydrogen density is derived at different region of our Galaxy. What are the principal sources of uncertainty in thus deriving the gas density? What important information is obtained from this study about the structure of the Galaxy? 2+2+1

Unit - 6

7. Answer ***any two*** questions :

(a) What do you mean by Ideal Momentum theory? Differentiate between propeller engine and jet engine. 3+2

(b) Write a note on the remarkable achievements of the Indian Space Research Organization. 5

(c) What is a viscous flow? Discuss the flow of a steady viscous incompressible conducting fluid through a rectangular duct with axis along the z-direction and the faces of the duct being parallel to the x and y directions respectively. 1+4

(d) Discuss the unsteady unidirectional motion of a semi-infinite mass of incompressible viscous electrically conducting fluid past an infinite plate. Analyze the problem mathematically. 5

[Throughout the paper take the Newton's Gravitational constant as $G = 6.67 \times 10^{-11}\ m^3kg^{-1}s^{-2}$]

DSE-B-2.3

(Advanced Mechanics)

Full Marks : 65

Symbols and Notations have their usual meanings unless otherwise stated.

Group - A

1. Answer the following multiple choice questions with only one correct option. Choose the correct option with proper justification : 2×10

(a) Two particles are connected by a rod of variable length $l = f(t)$. The nature of the constraint is

(i) scleronomic and holonomic (ii) rheonomic and non-holonomic

(iii) scleronomic and non-holonomic (iv) rheonomic and holonomic.

(b) Consider a planet of mass m orbiting around the sun under the inverse square law of attraction $\frac{\mu m}{r^2}$, $\mu > 0$. If the position of the planet at time t is given by the polar coordinate (r, θ), then the Lagrangian L of the system is given by

(i) $\frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - \frac{\mu m}{r}$ (ii) $\frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{\mu m}{r}$

(iii) $\frac{1}{2}m\left(\dot{r}^2 + \dot{\theta}^2\right) + \frac{\mu m}{r}$ (iv) $\frac{1}{2}m\left(\dot{r}^2 + \dot{\theta}^2\right) - \frac{\mu m}{r}$.

(c) The homogeneity of time leads to the law of conservation of

(i) linear momentum (ii) angular momentum

(iii) energy (iv) parity.

(d) The Lagrangian of a system is given by

$$L = \frac{1}{2}ml^2\left(\dot{\theta} + (\sin\theta)\dot{\varphi}^2\right) - mgl\cos\theta,$$

where m, l and g are constants.

Which of the following is conserved?

(i) $\dot{\varphi}\sin\theta$ (ii) $\frac{\dot{\varphi}}{\sin\theta}$

(iii) $\dot{\varphi}\sin^2\theta$ (iv) $\frac{\dot{\varphi}}{\sin^2\theta}$.

Please Turn Over

(e) The Hamiltonian corresponding to the Lagrangian $L = a\dot{x}^2 + b\dot{y}^2 - kxy$ is

(i) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} + kxy$ (ii) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - kxy$

(iii) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} + kxy$ (iv) $\frac{p_x^2 + p_y^2}{4ab} + kxy$.

(f) A particle moves in two dimensions with potential $V(x, y) = x + 2y$. Which of the following is a constant of motion?

(i) $p_y - 2p_x$ (ii) $p_x - 2p_y$

(iii) $p_x + 2p_y$ (iv) $p_y + 2p_x$.

(g) The Hamiltonian of a system is given by

$$H(q_1, q_2; \ p_1, p_2) = Kp_1^{\ 2} + \frac{K}{q_1^2} p_2^2 + \frac{l}{q_1},$$

where q_1, q_2 are generalised coordinates and p_1, p_2 are the generalised momenta and K, l are constants. Then

(i) $p_1 = Kt + l$ (ii) $p_2 = Kt + l$

(iii) p_1 is independent of time (iv) p_2 is independent of time.

(h) If in a scleronomic system the K.E. be a homogeneous function of velocity, then $\sum \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} - L$ will be equal to

(i) $T + V$ (ii) T

(iii) V (iv) $T - V$.

(The Symbols used have their usual meanings)

(i) A linear transformation of a generalised coordinate q and corresponding momenta p, to Q and P given by $Q = q + p$, $P = q + \alpha p$ is canonical if the value of the constant α is

(i) -1 (ii) 0

(iii) 1 (iv) 2.

(j) The Poisson bracket $[x, xp_y + yp_x]$ is equal to

(i) $-x$ (ii) y

(iii) $2p_x$ (iv) p_y.

Group - B

Unit - 1

(Marks : 10)

2. Answer ***any two*** questions :

(a) A particle of mass m moves in the field of force given by $\overline{F} = -\hat{r}\, k r \cos\phi$, where k is a constant and $\hat{r}$ is the radial unit vector. Find Lagrange's equation of motion and show that angular momentum is conserved. 3+2

(b) Define holonomic constraint. Give an example of a non-holonomic constraint.

The Lagrangian of a particle of unit mass is given by

$$L = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - V + A\dot{x} + B\dot{y} + C\dot{z},$$

where V, A, B and C are function of x, y, z.

Show that the equation of motion are

$$\ddot{x} = -\frac{\partial V}{\partial x} + \dot{y}\left(\frac{\partial B}{\partial x} - \frac{\partial A}{\partial y}\right) - \dot{z}\left(\frac{\partial A}{\partial Z} - \frac{\partial C}{\partial x}\right)$$

and two similar equations for $\ddot{y}$ and $\ddot{z}$. 1+1+3

(c) The energies of a dynamical system with 2 degrees of freedom are given by

$$2T = \frac{\dot{q}_1^2}{\left(a + bq_2^2\right)} + \dot{q}_2^2$$

and $\quad V = c + dq_2^2,$

where a, b, c, d are constants. Find q_2 by using idea of ignorable coordinates. 5

(d) A bead slides under uniform gravity on a wire in the shape of a cycloid given by

$$x = a\,(\theta - \sin\theta);\;\; y = a(1 + \cos\theta)\;\; 0 \leqslant 0 \leqslant 2\pi.$$

Find the Lagrangian function and the equation of motion. 4+1

Unit - 2

(Marks : 15)

3. Answer ***any three*** questions :

(a) If the Lagrangian of a particle of mass m constrained to move on the surface of a cylinder of radius 'a' and attracted towards the origin by a force proportional to the distance of the particle from the origin be expressed in cylindrical coordinate (r, θ, z) as

$L = \frac{1}{2} m\left(a^2 \dot{\theta}^2 + \dot{z}^2\right) - \frac{1}{2} K\left(a^2 + z^2\right)$, where K is constant, then

(i) find the Hamiltonian.

(ii) show that angular momentum about z-axis is a constant of motion. 3+2

(b) For a conservative holonomic system, state Hamilton's variational principle. Derive Lagrange's equation from this principle. 1+4

(c) State Noether's theorem. What do you mean by homogeneity of space? Show that homogeneity of space leads to the conservation of linear momentum. 1+1+3

(d) (i) What do you mean by Δ-variation of the path of a system? State principle of least action.

(ii) For a simple pendulum, obtain the expression of Hamiltonian function and derive Hamilton's canonical equations of motion. (1+1)+3

(e) The Hamiltonian of a dynamical system is given by $H = \frac{1}{2}\sum_{i=1}^{3}\left(p_i^2 + \mu^2 q_i^2\right)$,

where q_i, p_i are the generalised coordinates and momenta and μ is a constant.

Show that $F = q_2 p_3 - q_3 p_2$ is a constant of motion. 5

Unit - 3

(Marks : 10)

4. Answer ***any two*** questions :

(a) (i) Derive Hamilton's equations of motion from variational principle.

(ii) Suppose the Hamiltonian of a dynamical system is given by $H = \frac{1}{4}\left(p_1^{\,2} + p_2^{\,2}\right) - K q_1^{\,2}$,

where K is a constant.

Find the Lagrangian of the dynamical system. 3+2

(b) The kinetic energy T and potential energy V of a dynamical system are given by

$$T = \frac{1}{2}\left[A\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\psi}^2\right) + B\left(\dot{\psi}\cos\theta + \dot{\phi}\right)^2\right] \text{ and } V = mgl\cos\theta,$$

where A, B, l are arbitrary constants.

Construct the Hamiltonian of the dynamical system and also find Hamilton's equations of motion. 3+1+1

(c) What is an action of a mechanical system? State the principle of stationary action and hence show how does it lead to Fermat's principle. 1+1+3

(d) Show that in phase space area remains conserved under Hamiltonian flows. 5

Unit - 4

(Marks : 10)

5. Answer ***any two*** questions : 5×2

(a) (i) Show that the transformation $Q = q\tan p$, $P = \log(\sin p)$ is canonical.

(ii) Find the canonical transformation defined by the generating function

$$F_1(q, Q) = qQ - \frac{1}{2}m\omega q^2 - Q^2/(4m\omega),\ m,\ \omega \text{ are constants.}$$

(b) If u and v are two constants of motion in a given holonomic system, prove that the Poisson Bracket $[u, v]$ is also a constant of motion.

(c) Use Hamilton-Jacobi technique to solve the one-dimensional harmonic oscillator problem.

(d) Show that the transformation $Q = q\cos\theta - \frac{p}{\mu\omega}\sin\theta$, $P = \mu\omega q\sin\theta + p\cos\theta$ is a canonical transformation for all values of θ. Also find $[Q, P]_{(q, p)}$.