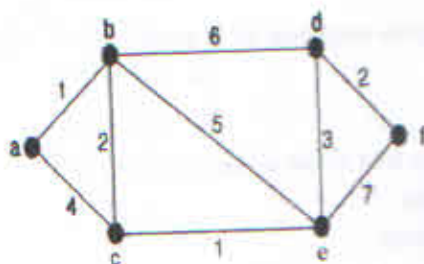


- (f) For $n \geq 1$, and positive integers a, b , if $\gcd(a, b) = 1$, then $\gcd(a^n, b^n)$ is
- (i) n (ii) na
(iii) nb (iv) 1.
- (g) Which one is a perfect number of the following numbers?
- (i) 21 (ii) 25
(iii) 28 (iv) 30.
- (h) The order of U_{20} is
- (i) 2 (ii) 4
(iii) 6 (iv) 8.
- (i) Value of $\phi(1024)$ is
- (i) 2^5 (ii) 2^8
(iii) 2^9 (iv) 2^{10} .
- (j) Which of the following integers has a primitive root?
- (i) 15 (ii) 16
(iii) 17 (iv) 18.

Unit - I

Answer *any five* questions.

2. If $m > \frac{(n-1)(n-2)}{2}$ in a simple graph G with n vertices and m edges, then prove that G is connected. 5
3. If G is a connected simple graph with no cycles, prove that G has at least two pendant vertices. 5
4. Prove that the graph $K_{3,3}$ is not a planar graph. Also prove if we delete any vertex from it, then it becomes a planar graph. 3+2
5. Apply Dijkstra's algorithm to the graph given below and find the shortest path from a to f . 5



(3)

B(5th Sm.)-Mathematics-H/DSE-B-1.1, DSE-B-1.2 &
DSE-B-1.3/CBCS

6. Prove that a tree with n vertices has $n-1$ edges. 5
7. Prove that a simple connected graph G is Eulerian if and only if degree of each vertex of G is even. 5
8. For the poset (P, \subseteq) , where $P = \{\phi, \{a\}, \{b\}, \{a, b\}\}$, determine (a) the greatest element, (b) the least element, (c) all maximal elements (d) all minimal elements. 1+1+1+2
9. Give an example of a graph which is both Eulerian and Hamiltonian. 5

Unit - II

Answer *any four* questions.

5×4

10. Find the remainder when $(n^2 + n + 41)^2$ is divided by 12.
11. Show that if p is a positive integer such that both p and $p^2 + 2$ are prime, then $p = 3$.
12. If the difference between two positive integers is n , then prove that their gcd divides n .
13. Prove that $3a^2 - 1$ is never a perfect square.
14. If ϕ is Euler-Totient function, then prove that $\phi(p^a) = p^a \left(1 - \frac{1}{p}\right)$ for any prime p and $gcd(a, p) = 1$.
15. Show that the cube of any natural number is of the form $7K$ or $7K \pm 1$.
16. If p be prime, then prove that $(p-1)! \equiv -1 \pmod{p}$.

Please Turn Over

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Paper : DSE-B-1.2
(Linear Programming and Game Theory)
Full Marks : 65

1. Answer the following multiple choice questions (MCQ) in which only one option is correct. Choose the correct option with proper justification if any. (1 mark for correct option and 1 mark for justification.)
2×10

- (a) Suppose that the objective function of an L.P.P. assumes its optimal value at more than one extreme point. Then
- (i) the value of the objective function will be different for different convex combinations of these extreme points.
 - (ii) the convex combination of these extreme points will improve the value of the objective function.
 - (iii) it indicates that the number of basic feasible solutions is degenerate.
 - (iv) every convex combination of these extreme points also gives the optimal value of the objective function.
- (b) Consider two sets $X = \{(x_1, x_2) \mid 2x_1 + x_2 \leq 4, x_1 + 4x_2 \geq 7, x_1, x_2 \geq 0\}$ and $Y = \{y \mid |y| \leq 3\}$. Then
- (i) X is a convex set, but Y is not a convex set
 - (ii) X is not a convex set, but Y is a convex set
 - (iii) none of X and Y are convex sets
 - (iv) both X and Y are convex sets.

- (c) The following L.P.P :

$$\begin{aligned} &\text{Maximize } Z = x_1 + x_2 \\ &\text{subject to } x_1 + x_2 \leq 1 \\ &\quad \quad \quad -3x_1 + x_2 \geq 3 \\ &\quad \quad \quad x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

has

- (i) one feasible solution
- (ii) two feasible solutions
- (iii) infinite number of feasible solutions
- (iv) no feasible solution.

Unit - II

3. Answer *any one* question :

(a) (i) Apply simplex method to solve the following L.P.P. :

$$\text{Maximize } Z = 3x_1 + 2x_2 + x_3$$

$$\text{subject to } 5x_1 + 6x_2 + 10x_3 \leq 50,$$

$$5x_1 + 4x_2 \leq 20,$$

$$15x_1 + 4x_2 \leq 30,$$

$$x_1, x_2, x_3 \geq 0.$$

(ii) Show that the L.P.P.

$$\text{Maximize } Z = 3x_1 + 2x_2$$

$$\text{subject to } 2x_1 + x_2 \leq 2,$$

$$3x_1 + 4x_2 \geq 12,$$

$$x_1, x_2 \geq 0.$$

has no feasible solution.

7+3

(b) (i) Solve the following L.P.P. by penalty method :

$$\text{Maximize } Z = -2x_1 + x_2 + 3x_3$$

$$\text{subject to } x_1 - 2x_2 + 3x_3 = 2,$$

$$3x_1 + 2x_2 + 4x_3 = 1,$$

$$x_1, x_2, x_3 \geq 0.$$

(ii) Use two phase simplex method to solve the following L.P.P. :

$$\text{Minimize } Z = x_1 + x_2$$

$$\text{subject to } 2x_1 + x_2 \geq 4,$$

$$x_1 + 7x_2 \geq 7,$$

$$x_1, x_2 \geq 0.$$

5+5

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Unit - III

4. Answer *any one* question :

(a) (i) Given the following L.P.P. :

$$\text{Maximize } Z = 3x_1 + 4x_2$$

$$\text{subject to } x_1 - x_2 \leq 1,$$

$$x_1 + x_2 \geq 4,$$

$$x_1 - 3x_2 \leq 3,$$

$$\text{and } x_1, x_2 \geq 0.$$

Using Simplex method, show that dual of the above L.P.P. has no feasible solution.

(ii) Formulate the dual of the L.P.P. given below :

$$\text{Maximize } Z = 2x_1 + 3x_2 + 4x_3$$

$$\text{subject to } x_1 - 5x_2 + 3x_3 = 7,$$

$$2x_1 - 5x_2 \leq 3,$$

$$3x_2 - x_3 \geq 5,$$

$$x_1, x_2 \geq 0$$

and x_3 is unrestricted in sign.

6+4

(b) (i) If the primal and its dual L.P.P. both have feasible solutions, then the objective function of both the L.P.P. are bounded.

[Bounded above for the maximization problem and bounded below for the minimization problem.]

(ii) Write down the dual of the L.P.P.

$$\text{Maximize } Z = 3x_1 + 4x_2$$

$$\text{subject to } x_1 + x_2 \leq 10,$$

$$2x_1 + 3x_2 \leq 18,$$

$$0 \leq x_1 \leq 8,$$

$$0 \leq x_2 \leq 6.$$

Find optimal solution of the above L.P.P. by solving its dual.

4+(2+4)

Unit - IV

5×3

5. Answer any three questions :

(a) Using VAM, solve the following transportation problem and find the optimal solution.

	D ₁	D ₂	D ₃	D ₄	a _j ↓
O ₁	19	20	50	10	7
O ₂	70	30	40	60	9
O ₃	40	8	70	20	18
b _j →	5	8	7	14	

(b) Find the optimal assignment from the following profit matrix :

	1	2	3	4
1	40	35	43	45
2	33	39	48	33
3	40	37	33	32
4	35	41	39	37

(c) A salesman has to visit five cities A, B, C, D, E. The distances (in hundred kilometers) between the cities are as follows :

	A	B	C	D	E
A	∞	3	6	2	3
B	3	∞	5	2	3
C	6	5	∞	6	4
D	2	2	6	∞	6
E	3	3	4	6	∞

If the salesman starts from city A and has to come back at city A, which route should he select so that the total distance travelled is minimum?

(d) Using Graphical method, solve the following rectangular game with the following pay-off matrix :

		Player B			
		B ₁	B ₂	B ₃	B ₄
Player A	A ₁	1	0	4	-1
	A ₂	-1	1	-2	5

Please Turn Over

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(e) Use dominance to reduce the pay-off matrix and then solve the game problem :

	B			
	-5	3	1	20
A	5	5	4	6
	-4	-2	0	-5

Paper : DSE-B-1.3

(Boolean Algebra and Automata Theory)

Full Marks : 65

1. All are multiple choice questions with single correct option. Students are required to opt the correct option and justify the correct option in their own words as far as practicable. One mark for correct option and one mark for justification. There is no negative marking. 2×10
- (a) A tree with N number of vertices contains
- | | |
|-------------------|----------------------|
| (i) $(N-1)$ edges | (ii) (N^2-1) edges |
| (iii) N edges | (iv) $(N+1)$ edges. |
- (b) Let $X = \{2, 3, 6, 12, 14\}$ and \leq be the partial order defined by $x \leq y$ if x divides y . Number of edges in the Hasse diagram of (X, \leq) is
- | | |
|---------|---------------------|
| (i) 3 | (ii) 4 |
| (iii) 5 | (iv) None of these. |
- (c) The complement of the expression $(W'X + YZ')$ is
- | | |
|----------------------------|-------------------------|
| (i) $(W' + X)(Y' + Z)$ | (ii) $(W + X')(Y' + Z)$ |
| (iii) $(W'' + X)(Y'' + Z)$ | (iv) $(W + X')(Y + Z')$ |
- (d) The following CFG
 $S \rightarrow As | bS | a | b$, is equivalent to the regular expression :
- | | |
|--------------------------|-------------------------|
| (i) $(a + b)^*$ | (ii) $(a + b)(a + b)^*$ |
| (iii) $(a + b)^*(a + b)$ | (iv) All of these. |
- (e) Regular expressions for all strings starts with ab and ends with bba is
- | | |
|----------------------|--------------------|
| (i) aba^*b^*bba | (ii) $ab(ab)^*bba$ |
| (iii) $ab(a+b)^*bba$ | (iv) All of these. |
- (f) The minimum number of productions required to produce a language consisting of palindrome string over $\Sigma = \{a, b\}$ is
- | | |
|---------|---------|
| (i) 3 | (ii) 5 |
| (iii) 7 | (iv) 6. |
- (g) The symmetric closure of the relation $\rho = \{(a, b) / a > b\}$ on the set of positive integers, is
- | | |
|-------------------------------|------------------------------|
| (i) $\{(a, b) / a \geq b\}$ | (ii) $\{(a, b) / a \leq b\}$ |
| (iii) $\{(a, b) / a \neq b\}$ | (iv) $\{(a, b) / a = b\}$. |

Please Turn Over

(0432+0441+0755)

Unit - IIAnswer *any one* question.

4. Simplify the logical function $y = x_1'x_2x_3' + x_1x_2x_3' + x_1'x_2x_3 + x_1x_2x_3$ by using Karnaugh map method, where x_1', x_3' are the complements of x_1 and x_3 respectively. 4
5. Construct a circuit diagram for the Boolean function— $(BC + A)(\overline{AB} + \overline{C}) + \overline{ABC}$.
Using laws of Boolean Algebra, simplify the function and draw the simplified circuit. 2+2

Unit - IIIAnswer *any one* question.

6. A Finite State Machine (FSM) is given as in the following diagram :



- (a) What is the output sequence for the given input data sequence?
0 0 1 0 1 0 1 1 0 1 1 0 1 1 1 1
- (b) What is the behaviour of the above finite state machine? 2+2
7. Let $M = (Q, \Sigma, \Gamma, q_0, \delta, \gamma)$ be a Finite State Machine with transition table

	δ		γ	
	q_2	q_1		
q_0	q_2	q_1	1	1
q_1	q_2	q_2	0	0
q_2	q_1	q_2	1	1

- (a) Draw the transition diagram of M .
- (b) What is the output string, if the input string is $ababab$? 2+2

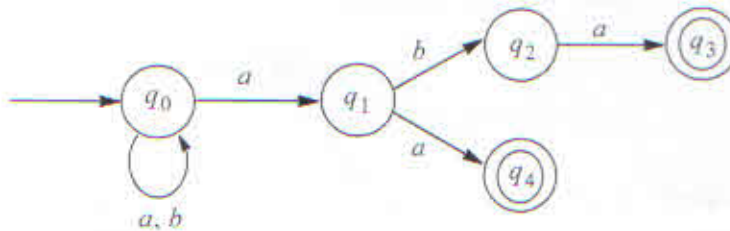
Unit - IVAnswer *any two* questions.

8. (a) Show that the language $L = \{0^{3m} 1^{2n} : m, n > 0\}$ is not regular using Pumping Lemma.
- (b) Construct a grammar which generates the language L described by the expression aa^*bb^* . 4+3

Please Turn Over

(0432+0441+0755)

9. Let M be the NDFA whose state diagram is given by the following figure :



- Write the transition table for M .
 - Find a directed walk with the label 'bbabbaa' from q_0 to q_4 .
 - What are the final states?
 - Is 'baa' in $L(M)$?
 - Find $\delta^*(q_0, baa)$
[where $L(M)$ and δ^* have their usual meanings.]
- 1+1+1+2+2
10. Let $G = (V_N, \Sigma, P, S)$ be a context-free grammar, where $V_N = \{S, A\}$, $\Sigma = \{a, b\}$, $P = \{S \rightarrow Ab, A \rightarrow aAb, A \rightarrow \lambda\}$
- Write a derivation of 'aaabbbb' from S in G .
 - Draw a derivation tree of the derivation of G with yield *aaaabbbb*.
 - Find the language $L(G)$ generated by this grammar.
- 2+2+3

Unit - V

Answer *any two* questions.

- Find a grammar in Chomsky Normal Form equivalent to the grammar $S \rightarrow aAbB, A \rightarrow aA|a, B \rightarrow bB|b$.
6
- Design a Turing Machine M , to recognize the language $\{1^n 2^n 3^n \mid n \geq 1\}$.
6
- Let $L = \{a^m b^n \mid n < m\}$ be a language. Construct—
 - a context-free grammar accepting L .
 - a push down automata accepting L by empty store.
 - a push down automata accepting L by final state.

2+2+2

Unit - VI

Answer *any one* question.

14. (a) Design the Turing machine accepting palindromes over $\{a, b\}$.
(b) Prove that union of two recursive languages is recursive. 3+4
15. (a) Solve the given PCP instance :
Top : $\{ab, b, a\}$
Bottom : $\{a, ba, b\}$
Determine if a match exists and explain your reasoning.
(b) Discuss why a general algorithm can not solve all PCP instances.
(c) Mention one field where undecidable problems like PCP pose a challenge. 4+2+1
-