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DERIVATION IN TERNARY SEMIRING

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Abstract: The purpose of this paper is to introduce the notion of derivation, differential ideal, differential k-ideal (differential h-ideal) in ternary semiring and to study the validity of some of the results on prime ideals and prime radicals of ternary semirings replacing ideals by differential ideals, radicals by differential radicals, ternary semirings by differential ternary semirings.Among others we deduce that a radical differential ideal I in a differential ternary semiring is the intersection of all prime differential ideals containing I. Also we deduce that in a differential ternary semiring satisfying ascending chain condition on radical differential ideals any radical differential ideal is expressible as the intersection of finite number of differential ideals.

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Key Words: ternary semiring, differential ternary semiring, differential ideal, differential k-ideal, differential h-ideal, radical differential ideal, radical differential k-ideal, radical differential h-ideal

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1. Introduction and Preliminaries

A nonempty set S together with a binary operation called addition and ternary multiplication denoted by juxtaposition, is said to be a *ternary semiring* [1] if S is an additive commutative semigroup satisfying the following conditions:

(i) $(abc)de = a(bcd)e = ab(cde)$,

(ii)$(a + b)cd = acd + bcd$,

(iii) $a(b + c)d = abd + acd$,

(iv) $ab(c + d) = abc + abd$, for all $a, b, c, d, e \in S$.

The set $\mathbb{Z}^-$ of all negative integers, with usual binary addition and ternary multiplication forms a ternary semiring.

This paper is sequel to our study of ternary semiring accomplished in [6]. The notion of ternary semiring was introduced by T. K. Dutta and S. Kar [1] in the year 2003, as a natural generalization of ternary ring, introduced by W. G. Lister [5] in 1971. In [6], among others, we have studied some important properties of prime radical of an ideal (a k-ideal and an h-ideal) in ternary semiring.Since ternary semiring is also a generalization of semiring, one of the important motivation of the present paper is [7]. The purpose of this paper is as stated in the abstract. We see that most of the results of [6] are valid in differential ternary semiring when ideal, k-ideal, h-ideal, prime ideal, prime radical are respectively replaced by differential ideal, differential k-ideal, differential h-ideal, differential prime ideal, differential prime radical.Among others, the important results are Theorems 2.30, 2.31,2.32, 2.33, 2.34. For most of the preliminaries we refer to [6] and its references. But some important notions are recalled at appropriate places.

2. Derivation in Ternary Semirings

Definition 2.1. A ternary semiring S is said to be *commutative* if $x_1x_2x_3 = x_{\sigma(1)}x_{\sigma(2)}x_{\sigma(3)}$ for all $x_1, x_2, x_3 \in S$ and $\sigma \in S_3$.

Example 1. Let T be the set of all continuous functions $f : X \to \mathbb{R}^-$ where X is a topological space and $\mathbb{R}^-$ is the set of all negative real numbers.

Now we define a binary addition and ternary multiplication on T by:

(i) $(f + g)(x) = f(x) + g(x)$

(ii) $(fgh)(x) = f(x)g(x)h(x)$, for all $f, g, h \in T$ and for all $x \in X$.

Then together with this binary addition and ternary multiplication T forms a ternary semiring and this ternary semiring is commutative.

Definition 2.2. [1] Let S be a ternary smiring. If there exists an element $0 \in S$ such that $0 + x = x$ and $0xy = x0y = xy0 = 0$ for all $x, y \in S$, then '0' is called the *zero element* or simply the *zero* of the ternary semiring S. In this case we say S is a ternary semiring with zero.

Example 2. $\mathbb{Z}_0^-$, the set of all negative integers with zero forms a ternary semiring with zero element '0', where the usual addition, usual multiplication of integers are respectively the binary operation and ternary operation.

Definition 2.3. [1] Let S be a ternary semiring. If there exists an element $e \in S$ such that $eex = exe = xee = x$ for all $x \in S$, then 'e' is called a *unital element* of the ternary semiring S.

Example 3. Let T be the set of all real numbers and k be a fixed number in T. If we define a binary addition and ternary multiplication on T respectively by $a + b = 0$ and $abc = a + b + c + k$ for all $a, b, c \in T$, then with this binary addition and ternary multiplication, T is a ternary semiring with $\frac{-k}{2}$ as a unital element.

Throughout this paper unless otherwise stated a ternary semiring means commutative ternary semiring with zero.

Definition 2.4. Let S be a ternary semiring. A mapping $d : S \to S$ ($d(a)$ is denoted by a' for all $a \in S$) is said to be a derivation on S if
(i) $(a_1 + a_2)' = a_1' + a_2'$ for all $a_1, a_2 \in S$,
(ii) $(a_1a_2a_3)' = a_1'a_2a_3 + a_1a_2'a_3 + a_1a_2a_3'$ for all $a_1, a_2, a_3 \in S$,
(iii) $0' = 0$,
(iv) $e' = 0$, provided S has unital element e.

We call $d(a)$, that is a' the derivative of a.

Note. For every ternary semiring S there exists a derivation d on S, viz, $d(s) = 0$ for all $s \in S$. This derivation is called the trivial derivation.

Definition 2.5. A ternary semiring together with a non trivial derivation is said to be a *differential ternary semiring.*

Example 4. Let $U = \left\{ \begin{bmatrix} a & b \\ 0 & a \end{bmatrix} : a, b \in \mathbb{Z}_0^- \right\}$. Then U is a ternary semiring with respect to usual matrix addition and ternary matrix multiplication. We define $d : U \mapsto U$ by $\begin{bmatrix} a & b \\ 0 & a \end{bmatrix} \mapsto \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$. Then we can easily verify that d is a nontrivial derivation on U. Thus U becomes a differential ternary semiring.

The following result can be easily deduced using definition of derivation where trivial derivation acts as identity.

Proposition 2.6. *Let S be a ternary semiring and $\mathcal{D}$ denote the set of all derivations on S. We define $(D_1 + D_2)(x) = D_1(x) + D_2(x)$ for $x \in S$ and $D_1, D_2 \in \mathcal{D}$. Then $(\mathcal{D}, +)$ is a monoid.*

Theorem 2.7. *In any differential ternary semiring S, the elements with derivative zero forms a ternary subsemiring of S.*

Proof. let $C = \{x \in S : x^{'} = 0\}$.
Clearly $0 \in C$. So C is nonempty.
Let $a, b, c \in C$.
Then $a^{'} = 0, \quad b^{'} = 0, \quad c^{'} = 0$.
Now $(a + b)^{'} = a^{'} + b^{'} = 0 + 0 = 0$.
and $(abc)^{'} = a^{'}bc + ab^{'}c + abc^{'} = 0bc + a0c + ab0 = 0 + 0 + 0 = 0$.
Thus $a + b \in C$ and $abc \in C$.
Hence C is a ternary subsemiring of S. □

Definition 2.8. [4] An additive subsemigroup I of a ternary semiring S is called a *left (right, lateral) ideal* of S if $s_1 s_2 i$ (respectively $i s_1 s_2$, $s_1 i s_2$) $\in I$ for all $s_1, s_2 \in S$ and $i \in I$. If I is a left, a right, a lateral ideal of S, then I is called an *ideal* of S.

Example 5. Let $M_2(\mathbb{Z}_0^-)$ be the ternary semiring of all 2×2 square matrices over $\mathbb{Z}_0^-$ and $I = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in 2\mathbb{Z}_0^- \right\}$. Then I is an ideal of $M_2(\mathbb{Z}_0^-)$.

Definition 2.9. [1] An ideal I of a ternary semiring S is called a *k-ideal* if $x + y \in I$, $x \in S$, $y \in I$ imply that $x \in I$.

Example 6. In the ternary semiring $\mathbb{Z}_0^-$ of all non negative integers with zero, the subset $P = \left\{3k : k \in \mathbb{Z}_0^-\right\}$ is a k-ideal.

Definition 2.10. [1] An ideal I of a ternary semiring S is called an *h-ideal* if $x + y_1 + z = y_2 + z$, $x, z \in S$ and $y_1, y_2 \in I$ imply $x \in I$.

Definition 2.11. If d is a nontrivial derivation on a ternary semiring S then an ideal I of S is said to be a *d-differential ideal* of S if $a^{'} \in I$ whenever $a \in I$.

We simply write differential ideal instead of d-differential ideal.

Definition 2.12. A k-ideal (h-ideal) which is also a differential ideal is said to be a *differential k-ideal (h-ideal).*

Example 7. With same notation as in Example 4 we take
$I = \left\{ \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} : a \in \mathbb{Z}_0^- \right\}.$
Then I is a differential ideal of U. In fact here $I^{'} = I$.

It is easy to deduce the following.

Theorem 2.13. *In a differential ternary semiring intersection of any collection of differential ideals (differential k-ideals, differential h-ideals) is again a differential ideal (differential k-ideal, differential h-ideal).*

Definition 2.14. [1] An equivalence relation ρ on a ternary semiring S is said to be a *congruence* on S if the following conditions hold:
(i) $a\rho a_1$ and $b\rho b_1 \Rightarrow (a+b)\rho(a_1+b_1)$ (ii) $a\rho a_1, b\rho b_1, c\rho c_1 \Rightarrow (abc)\rho(a_1b_1c_1)$ for all $a, b, c, a_1, b_1, c_1 \in S$.

Definition 2.15. [1] Let I be a proper ideal of a ternary semiring S. Then the congruence on S denoted by ρ_1 and defined by $s\rho_1 s_1$ if and only if $s + a_1 = s_1 + a_2$ for some $a_1, a_2 \in I$ and $s, s_1 \in S$ in called *Bourne congruence* on S defined by the ideal I.

The *Bourne congruence class* of an element $s \in S$ is denoted by s/ρ_1 or simply by s/I and the set of all such congruence classes of S is denoted by S/ρ_1 or simply by S/I.
Remark. It should be noted that for any proper ideal I of S and s/I is not necessarily equal to $s + I = \{s + a : a \in I\}$ but surely contains it.

Definition 2.16. [1] For any proper ideal I of a ternary semiring S if the Bourne congruence ρ_1 defined by I is proper then we can define addition and ternary multiplication in S/I by $(a/I + b/I) = (a+b)/I$ and $(a/I)(b/I)(c/I) = (abc)/I$ for all $a, b, c \in S$. With these two operations S/I forms a ternary semiring which is called the *Bourne factor ternary semiring* or simply the *factor ternary semiring.*

Theorem 2.17. *Let S be a ternary differential semiring and I be a ideal of S. Then the Bourne factor ternary semiring S/I is a differential ternary semiring.*

Proof. We define a mapping $d : S/I \to S/I$ by $d(s/I) = s^{'}/I$, where $s^{'}$ is the derivative of s in S.
Let $s_1, s_2 \in S$.
Then $d(s_1/I + s_2/I) = d((s_1 + s_2)/I)$
$= (s_1 + s_2)^{'}/I$
$= (s_1^{'} + s_2^{'})/I$

$$= s_1'/I + s_2'/I$$
$$= d(s_1/I) + d(s_2/I).$$
$$d((s_1/I)(s_2/I)(s_3/I)) = d(s_1s_2s_3/I)$$
$$= (s_1s_2s_3)'/I$$
$$= (s_1's_2s_3 + s_1s_2's_3 + s_1s_2s_3')/I$$
$$= (s_1's_2s_3/I) + (s_1s_2's_3/I) + (s_1s_2s_3'/I)$$
$$= (s_1'/I)(s_2/I)(s_3/I) + (s_1/I)(s_2'/I)(s_3/I) + (s_1/I)(s_2/I)(s_3'/I)$$
$$= d(s_1/I)(s_2/I)(s_3/I)+(s_1/I)d(s_2/I)(s_3/I)+(s_1/I)(s_2/I)d(s_3/I)$$

So d is a derivation on S/I.
Hence S/I is a differential ternary semiring. □

Definition 2.18. [2] A proper ideal P of a ternary semiring S is called a prime ideal of S if for any three ideals A, B, C of S, $ABC \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$.

Definition 2.19. [3] The *prime radical* of an ideal I in a ternary semiring S is denoted by $\beta(I)$ and is defined to be the intersection of all prime ideals of S each of which contains I.

Proposition 2.20. [6] *In a commutative ternary semiring S, the prime radical of an ideal I, $\beta(I) = \{x \in S : x^{2n+1} \in I$, where n is an integer and $n \geq 0$ $\}$.*

Definition 2.21. [6] An ideal I in a ternary semiring S is said to be a *prime radical ideal* if $\beta(I) = I$.

In what follows, for simplicity, we write radical ideal instead of prime radical ideal.

Definition 2.22. A radical ideal (k-ideal, h-ideal) which is also a differential ideal is called a radical differential ideal (k-ideal, h-ideal).

Theorem 2.23. *If abc lies in a radical differential k-ideal I of a ternary semiring S, then $a'bc \in I, ab'c \in I, abc' \in I$.*

Proof. Let $abc \in I$. Then $(abc)' \in I$ i.e., $(a'bc + ab'c + abc') \in I$. Which implies $abc'abc'(a'bc+ab'c+abc') \in I$ i.e., $(abc')^2a'bc+(abc')^2ab'c+(abc')^3 \in I$. Now $(abc')^2a'bc \in I$, $(abc')^2ab'c \in I$. Hence $(abc')^3 \in I$ (cf. Definition 2.9). Consequently, in view of Proposition 2.20, $abc' \in I$. Similarly we can prove $a'bc \in I, ab'c \in I$. □

We check the validity of Theorem 2.14, Theorem 2.15 of [6] in the present content.

Theorem 2.24. *Let I be a radical differential k-ideal of a ternary semiring S and U, V are any two subsets of S. Then*

$$T = \{x \in S : xUV \subseteq I\}$$

is a radical differential k-ideal.

Proof. Clearly T is an ideal of S.

Let $x \in T$. Then $xuv \in I$, for all $u \in U, v \in V$. Then by Theorem 2.23, $x^{'}uv \in I$, $xu^{'}v \in I$, $xuv^{'} \in I$. Hence $x^{'}uv \in x^{'}UV \subseteq I$. So $x^{'} \in T$. Consequently, T is a differential ideal of S. The rest of the proof is similar to that of Theorem 2.14 [6]. □

The following is the h-ideal analogue of Theorem 2.24 whose proof is omitted as it follows in a similar fashion as that of Theorem 2.24.

Theorem 2.25. *Let I be a radical differential h-ideal of a ternary semiring S and U, V are any two subsets of S. Then*

$$T = \{x \in S : xUV \subseteq I\}$$

is a radical differential h-ideal.

Now by using Proposition 2.20, Definition 2.9, 2.10 and 2.21 we deduce the following result.

Proposition 2.26. *In a differential ternary semiring S, the intersection of any collection of radical differential k-ideals (h-ideals) is again a radical differential k-ideal (respectively h-ideal).*

Definition 2.27. [1] A nonempty subset A of ternary semiring S is called an *m-system* if for each $a, b, c \in A$ there exist elements x_1, x_2, x_3, x_4 of S such that $ax_1bx_2c \in A$ or $ax_1x_2bx_3x_4c \in A$ or $ax_1x_2bx_3cx_4 \in A$ or $x_1ax_2bx_3x_4c \in A$.

Example 8. Let $S = \left\{ \begin{bmatrix} a & b \\ 0 & a \end{bmatrix} : a, b \in \mathbb{Z}_0^- \right\}$ and $A = \left\{ \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} : a \in \mathbb{Z}_0^- \right\}$. Then A is an m-system.

Definition 2.28. Let S be a ternary semiring and A be any subset of S. The differential ideal generated by A is denoted by $\{A\}$ and defined to be intersection of all differential ideals of S which contains A.

For simplicity we write $\{A \cup \{a\}\}$ as $\{A, a\}$.

Note. In order to avoid any possible confusion we note here that in [1], $\{A, a\}$ denotes radical ideal generated by $\{A \cup \{a\}\}$.

Theorem 2.29. *Let P be an m-system in a differential ternary semiring S and I be differential ideal of S which does not meet P. Then I is contained in a maximal differential ideal Q of S which does not meet P and such Q is prime.*

Proof. Let $\mathcal{U}$ be the set of all differential ideals of S containing I and none of which meets P.

Then $\mathcal{U}$ is a poset with respect to set inclusion relation. Then Zorn's lemma ensures that $\mathcal{U}$ has a maximal element Q (say).

Therefore $P \cap Q = \phi$. If possible let Q is not prime, then there exists $a, b, c \in S$ such that $a \notin Q$, $b \notin Q$, $c \notin Q$ but $abc \in Q$. Then $\{Q, a\}$, $\{Q, b\}$, $\{Q, c\}$ are differential ideals properly containing Q.

Then by maximality of Q, $\{Q, a\} \cap P \neq \phi$, $\{Q, b\} \cap P \neq \phi$ and $\{Q, c\} \cap P \neq \phi$.

Let $p_1 \in \{Q, a\} \cap P$, $p_2 \in \{Q, b\} \cap P$, $p_3 \in \{Q, c\} \cap P$. Since P is an m-system there exist $s_1, s_2, s_3, s_4 \in S$ such that $p_1 s_1 p_2 s_2 p_3 \in P$ or $p_1 s_1 s_2 p_2 s_3 s_4 p_3 \in P$ or $p_1 s_1 s_2 p_2 s_3 p_3 s_4 \in P$ or $s_1 p_1 s_2 p_2 s_3 s_4 p_3 \in P$. If $p_1 s_1 p_2 s_2 p_3 \in P$ then $p_1 s_1 p_2 s_2 p_3 \in \{Q, a\}\{Q, b\}\{Q, c\} \subseteq \{Q, abc\} \subseteq Q$, which contradicts the fact that $P \cap Q = \phi$. If $p_1 s_1 s_2 p_2 s_3 s_4 p_3 \in P$ then $p_1 s_1 s_2 p_2 s_3 s_4 p_3 \in \{Q, a\}\{Q, b\}\{Q, c\} \subseteq \{Q, abc\} \subseteq Q$, which contradicts the fact that $P \cap Q = \phi$. If $p_1 s_1 s_2 p_2 s_3 p_3 s_4 \in P$ then $p_1 s_1 s_2 p_2 s_3 p_3 s_4 \in \{Q, a\}\{Q, b\}\{Q, c\} \subseteq \{Q, abc\} \subseteq Q$, which contradicts the fact that $P \cap Q = \phi$. If $s_1 p_1 s_2 p_2 s_3 s_4 p_3 \in P$ then $s_1 p_1 s_2 p_2 s_3 s_4 p_3 \in \{Q, a\}\{Q, b\}\{Q, c\} \subseteq \{Q, abc\} \subseteq Q$, which contradicts the fact that $P \cap Q = \phi$. Thus in any case we arrive at a contradiction.

Consequently, Q is prime. □

Theorem 2.30. *Let I be a radical differential ideal in a differential ternary semiring S. Then I is the intersection of all prime differential ideals containing I.*

Proof. Let B be the intersection of all prime differential ideals in S each of which contains I. Clearly $I \subseteq B$. Let $x \notin I$ and $P = \{x^{2n+1} : n \in \mathbb{Z}_0^+\}$. Then P is an m-system and $P \cap I = \phi$.

Then by Theorem 2.29, there exists a maximal differential ideal $Q \supseteq I$ such that Q does not meet P and Q is prime. Therefore $P \cap Q = \phi$. Now $x \in P$, by construction of P so $x \notin Q$. Hence $x \notin B$. Hence $B \subseteq I$. Consequently, $B = I$. □

Now we obtain below the analogue of Theorem 2.21 in [6] in the setting of differential ternary semiring.

Theorem 2.31. *In a differential ternary semiring S satisfying ascending chain condition on radical differential ideals any radical differential ideal is expressible as the intersection of finite number of prime differential ideals.*

Proof. Let S be a differential ternary semiring satisfying ascending chain condition on radical ideals.

Let X be the set of all radical differential ideals which cannot be expressed as the intersection of finite number of prime differential ideals.

As S satisfies ascending chain condition on radical differential ideals, X has a maximal element say I.

Since I is a radical differential ideal and it cannot be expressed as an intersection of finite number of prime differential ideals, I is not prime. Therefore there exist $a, b, c \in S$ such that $abc \in I$ but $a \notin I$, $b \notin I$, $c \notin I$. Then $\{I, a\}$, $\{I, b\}$, $\{I, c\}$ are radical differential ideals in S, each of which properly contains I. Therefore $\{I, a\}$, $\{I, b\}$, $\{I, c\}$ are expressible as intersection of finite number of prime differential ideals. Now $\{I, a\}\{I, b\}\{I, c\} \subseteq \{I, abc\} \subseteq I$. Let $d \in \{I, a\} \cap \{I, b\} \cap \{I, c\}$. Then $d^3 \in \{I, a\}\{I, b\}\{I, c\} \subseteq I$. Then in view of Proposition 2.20 and Definition 2.21, $d \in I$. So $\{I, a\} \cap \{I, b\} \cap \{I, c\} \subseteq I$. Hence $I = \{I, a\} \cap \{I, b\} \cap \{I, c\}$, which is a contradiction. This completes the proof. □

Theorem 2.32. *Let S be a differential ternary semiring with a differential ternary subsemiring A and I be a differential ideal in S such that $P = I \cap A$ is a prime differential ideal in S. Then I can be enlarged to a prime differential ideal J in S which also contracts to P i.e. there exists a prime differential ideal J in S such that $I \subseteq J$ and $P = J \cap A$.*

Proof. Let Q be the complement of P in A. Then Q is an m-system in A. So Q is an m-system in S and $Q \cap I = \phi$. Then by Theorem 2.29, there exists a maximal differential ideal J in S which contains I and does not meet Q and J is prime. Hence $P = I \cap A \subseteq J \cap A$. Let $x \in J \cap A$. Then $x \notin Q$. So $x \in P$. Hence $J \cap A \subseteq P$. Consequently, $P = J \cap A$. □

The following is the analogue of Theorem 2.19 of [6].

Theorem 2.33. *Let S be a differential ternary semiring and A be a differential ternary subsemiring of S. Let I be a radical differential ideal of S such that $abc \in I$, $a \in A$, $b, c \in S$ imply either $a \in I$ or $b \in I$ or $c \in I$. Then $P = I \cap A$ is a prime differential ideal in A. Also I can be expressed as an intersection of prime differential ideals each of which contracts to P.*

Proof. That P is a prime ideal follows from Theorem 2.19 [6].

Now let $a \in P$. Then I and A both being differential ideals $a^{'} \in I$ and $a^{'} \in A$. Hence $a^{'} \in P$. Consequently P is a differential ideal in A.

The proof of the last part follows in the manner similar to that of Theorem 2.19 [6]. □

Now combining Theorem 2.32 and first part of Theorem 2.33, we obtain the following theorem:

Theorem 2.34. *Let S be a differential ternary semiring with a differential ternary subsemiring A and I be a differential ideal in S such that $abc \in I$, $a \in A$, $b, c \in S$ imply either $a \in I$ or $b \in I$ or $c \in I$. Then $P = I \cap A$ is a prime differential ideal in A and I can be enlarged to a prime differential ideal in S which also contracts to P.*

Concluding Remark. There are plenty of works on derivations in rings as well as in semirings. Ternary semiring being a generalization of ring as well as of semiring,there is further scope of studying derivation in ternary semiring.

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